Mark Scheme (Results)

Summer 2017

Pearson Edexcel International A Level
In Mechanics M3 (WME03) Paper 1

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $*$ The answer is printed on the paper
- $\lfloor$ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. Be careful when scoring a response that is either all correct or all incorrect. It is very easy to click down the '0' column when it was meant to be '1' and all correct.

General Principles for Mechanics Marking

Usual rules for M marks: correct no. of terms; dim correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
Omission or extra g in a resolution is accuracy error not method error.
Omission of mass from a resolution is method error.
Omission of a length from a moments equation is a method error.
Omission of units or incorrect units is not (usually) counted as an accuracy error.
DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.
Use of g = 9.81 should be penalised once per (complete) question.
N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *ONCE* per complete question.
However, premature approximation should be penalised every time it occurs.

In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.

Accept column vectors in all cases.

M3 WME03 June 2017
Mark Scheme

Question Number	Scheme	Marks
1	$\text{Vol} = \int_2^4 (\pi) \times \frac{1}{4} x \, \mathrm{d}x$	
	$= (\pi)\left[\frac{1}{8}x^2\right]_2^4$	M1
	$= (\pi)\left[2 - \frac{1}{2}\right] = \frac{3(\pi)}{2}$	A1
	$\int_2^4 (\pi) \times \frac{1}{4} x^2 \mathrm{d}x$	
	$= (\pi)\left[\frac{1}{12}x^3\right]_2^4$	M1
	$= (\pi)\frac{1}{12}[64-8] = \frac{56}{12}(\pi)$	A1
	$\bar{x} = \frac{56}{12}\pi \times \frac{2}{3\pi} = \frac{28}{9}$	M1A1 (6)

NB Centre of mass of a LAMINA scores 0/6
For the first 4 marks π not needed in either integral. For the third M mark, π must be included in both integrals or neither.

M1 Use $\text{Vol} = (\pi)\int_2^4 y^2 \, \mathrm{d}x$ and attempt the integration. Limits not needed.

A1 Correct volume following substitution of correct limits. Can be decimal or implied by a correct final answer.

M1 Use $(\pi)\int_2^4 xy^2 \, \mathrm{d}x$ and attempt the integration. Limits not needed.

A1 Correct result following substitution of correct limits. Can be decimal or implied by a correct final answer.

M1 Use $\dfrac{\int \pi x y^2 \mathrm{d}x}{\int \pi y^2 \mathrm{d}x}$ (with their values for the integrals)

A1 Correct x coordinate. Must be exact. Give A0 if decimal equivalent of 56/12 is given.

Question Number	Scheme	Marks
2(a)	Mass $\frac{1}{3}\pi\times16r^2\times4h$ $\frac{1}{3}\pi\times9r^2\times3h$ $\frac{1}{3}\pi\times37r^2h$	B1
	(Ratio 64 27 37)	
	Dist h $h+\frac{3}{4}h$ $\bar{x}$	B1
	$64h-27\times\frac{7}{4}h=37\bar{x}$	M1A1ft
	$\bar{x}=\frac{67}{148}h$ accept $0.45(270.....)h$ or better	A1 (5)
(b)	$\tan\theta=\frac{h-\bar{x}}{3r},=\frac{1-\frac{67}{148}}{3}$	M1,A1ft
	$\theta=\tan^{-1}\left(\frac{148-67}{3\times148}\right)$ $(=10.34...)$	
	Reqd angle $=\tan^{-1}\left(\frac{r}{3r}\right)-\theta=8.096...°$	M1A1 (4)
	Accept 8.1 or better	[9]
ALT:	$\tan\theta=\frac{3r}{h-\bar{x}},=\frac{3}{1-\frac{67}{148}}$	M1A1
	Reqd angle $=\theta-\tan^{-1}(3)=8.096...°$	M1A1

(a)

B1 Correct ratio of masses, any equivalent. (Mark the ratio, not formulae.)

B1 Correct distances from *O* or any other point

M1 Use their mass ratio and distances to form a moments equation with 3 terms.

A1ft Correct equation, follow through their mass ratio and distances.

A1 Correct distance from *O*. Exact or min 2 sf

(b)

M1 Use their $\bar{x}$ to form an expression for $\tan\theta$. Can include h and r. $h-\bar{x}$ needed but fraction can be either way up. Denom (numerator in ALT) can be r, $3r$ or $4r$.

A1ft Fraction either way up and $h = r$ used. Follow through their $\bar{x}$

M1 Complete the method to obtain the required angle

A1 Correct size of the angle, 2 sf min. Radians accepted - 0.14 or better.

NB If solid is hung from a point on the rim of the base give M1A0M0A0

Question Number	Scheme	Marks
3(a)	$\frac{2\pi}{\omega} = 0.25, \quad \omega = 8\pi$	M1, A1
	max accel $= a\omega^2 = 0.25 \times 64\pi^2 \ \ = 16\pi^2 \quad (158)$ m s^{-2}	dM1A1 (4)
(b)	$\omega_1 = 8\pi, \quad a_1 = 0.125$ accept $\frac{0.25}{2}$	B1ft
	Max speed for the new motion $= 8\pi \times 0.125$ m s^{-1} $(= \pi)$	M1(either)
	Max speed for the original motion $= 8\pi \times 0.25$ m s^{-1} $(= 2\pi)$	A1ft(both)
	$\lvert I \rvert = 0.5(8\pi \times 0.25 + 8\pi \times 0.125) = 1.5\pi$ Ns $(= 4.7123...$ accept 4.7 or better$)$	dM1A1 (5)
		[9]

(a)

M1 Use $\frac{2\pi}{\omega} = \frac{1}{4}$ or 4 to obtain a value for ω

A1 Correct value of ω exact or decimal

dM1 Use $a\omega^2$ with their ω and a = 0.25 to obtain the max magnitude of the acceleration. Depends on the first M mark.

A1 Correct max magnitude, exact or 158 or better

(b)

B1ft New ω and amp, follow through their original ω and amp

M1 One speed needed for this mark (but 2 for the complete problem). Award M1 for either, using their ω and amp. May obtain v or v^2.

A1ft Award A1 if both speeds are correct, follow through their ω and amp.

dM1 Use impulse = change of momentum, with their speeds (neither = 0), to obtain the magnitude of the impulse. Depends on the first M mark of this part. Allow if momenta are subtracted as long as no incorrect formula seen.

A1 Correct magnitude, exact or min 2 sf. Must be positive.

Question Number	Scheme	Marks
4 (a)	$0.3g\left(x+0.4\right)=\dfrac{49x^2}{2\times0.4}$ OR $0.3gy=\dfrac{49\left(y-0.4\right)^2}{2\times0.4}$	M1A1A1
	$5x^2-0.24x-0.096=0$	dM1
	$x=\dfrac{0.24\pm\sqrt{0.24^2+20\times0.096}}{10}$	
	$x=0.1646....\left(\text{neg not needed}\right)$ $y=0.5646...$ (0.24 need not be shown)	dM1
	$AB=0.56$ or $0.565\,\text{m}$ $AB=y=0.56$ or 0.565	A1 (6)
(b)	$\dfrac{49\times0.2^2}{0.8}=\dfrac{1}{2}\times0.3v^2+0.3g\times0.6$	M1A1A1
	$v^2=\dfrac{2}{0.3}\left(\dfrac{49\times0.2^2}{0.8}-0.3\times9.8\times0.6\right)$	
	$v=2.1$ or $2.14\ \text{m s}^{-1}$	dM1A1 (5) [11]

(a)

M1 Use an energy equation with x as the extension at B or y as the distance fallen. There must be a PE term and an EPE term. EPE term to be of the form $k\dfrac{\lambda x^2}{l}$

A1A1 Deduct one mark per incorrect term.

dM1 Simplify to a 3 term quadratic, terms in any order. Depends on first M mark

M1 Solve their quadratic by formula or completing the square. Allow calculator solution **only** if $x=0.1646$ or the final answer is correct. Depends on both previous M marks.

A1 Correct length of AB. Must be 2 or 3 sf.

ALT: Find v at natural length by SUVAT and then use energy. No marks until the energy equation seen, then mark as above. (A1A1 deduct one per error.)

(b)M1 Forming an energy equation from release to A. Must have 3 terms, an initial EPE, a PE and a KE term. EPE term to be of the form $k\dfrac{\lambda x^2}{l}$ and extension $\neq 0.165$

A1 Any two terms correct.

A1 Completely correct equation.

dM1 Solve their equation to $v^2 =$ (4.5733...) or $v =$ ... Depends on the M mark above (in (b))

A1 Speed = 2.1 or 2.14 m s^{-1}

NB Use of $g = 9.81$ produces the same 3 sf answers. Exceptionally allow this.
SHM solutions must first **prove** SHM and find the centre (equilibrium position). Send to review

Question Number	Scheme	Marks
5(a)	$0.4\ddot{x} = -\dfrac{k}{x^2}$	M1
	$0.4v\dfrac{\mathrm{d}v}{\mathrm{d}x} = -\dfrac{k}{x^2}$	M1
	$0.2v^2 = \int -kx^{-2}\mathrm{d}x$	
	$0.2v^2 = \dfrac{k}{x}\,(+c)$	dM1A1ft
	$x=2,\ v=5 \Rightarrow 5 = \dfrac{k}{2}+c$	dM1
	$x=5,\ v=2 \Rightarrow 0.8 = \dfrac{k}{5}+c$	A1
	$4.2 = k\left(\dfrac{1}{2}-\dfrac{1}{5}\right)$	dM1
	$k=14$	A1 cso (8)
(b)	$c = 5-7 = -2$	
	$0.2v^2 = \dfrac{14}{x} - 2$	M1A1ft
	$v=0 \Rightarrow x=7$	dM1A1 cso(4) [12]

(a)M1 Form an equation of motion, minus sign may be missing.

M1 Writing the acceleration in the form $v\dfrac{\mathrm{d}v}{\mathrm{d}x}$ These two M marks may be awarded together.

Can be implied by $\dfrac{1}{2}mv^2$ after integrating.

dM1 Attempt to integrate both sides of the equation wrt x Depends on both M marks above

A1ft Correct integration with correct signs. Constant may be missing. Follow through a missing minus sign.

NB For the first 4 marks m or 0.4 may be used

dM1 Substitute either $x=2,\ v=5$ or $x=5,\ v=2$ Depends on all M marks above.

A1 Both substitutions made and 2 correct equations in k and c found

dM1 Solve these simultaneous equations to obtain a value for k. Solving 1 linear equation (as c was omitted) scores M0. Depends on all M marks above.

A1 Correct value of k obtained.

(b)

M1 Obtain a value of c and form an expression for v^2. (Often seen in (a); award marks if (b) is attempted.)

A1ft Correct expression for v^2. Follow through $k=-14$ which gives $c=-5$

dM1 Substitute $v = 0$ in their expression for v^2 and solve for x

A1cso Correct value of x obtained.

Question Number	Scheme	Marks
6 (a)	$\mathrm{R}(\uparrow)\quad R\cos\theta = mg$	B1
	$\mathrm{R}(\rightarrow)\quad R\sin\theta = m\dfrac{v^2}{50}$	M1A1
	$\tan\theta = \dfrac{3}{4} = \dfrac{v^2}{50g}$	
	$v^2 = \dfrac{150g}{4}\quad v = 19.17... = 19 \text{ or } 19.2 \text{ m s}^{-1}$	dM1A1 (5)
(b)	$\mathrm{R}(\uparrow)\quad R\cos\theta - F\sin\theta = mg$	M1A1
	$\mathrm{R}(\rightarrow)\quad R\sin\theta + F\cos\theta = m\dfrac{v^2}{50}$	M1A1A1
	$F = \dfrac{1}{4}R$	B1
	$\dfrac{4}{5}R - \dfrac{1}{4}R \times \dfrac{3}{5} = mg$	
	$\dfrac{3}{5}R + \dfrac{1}{4}R \times \dfrac{4}{5} = m\dfrac{v^2}{50}$	
	$\dfrac{v^2}{50g} = \dfrac{4}{5} \div \dfrac{13}{20},\quad v = 24.55.... = 25 \text{ or } 24.6 \text{ m s}^{-1}$	dM1,A1 (8)
		[13]

(a)B1 Resolve vertically, equation must be fully correct.

M1 Form an equation of motion horizontally. *R* must be resolved; acceleration can be in either form.

A1 Correct equation, acceleration $v^2/50$ or v^2/r.

ALT Equation parallel to the track: $mg\sin\theta = m\dfrac{v^2}{50}\cos\theta$

M1 weight and acceleration both resolved A1A1 one mark each term (50 or *r*)
No B mark here.

dM1 Use $\tan\theta = 3/4$ with the eqns (or eqn) to reach v^2 = ... or v = ... Depends on the M1 above.

A1 Correct value of *v*. Must be 2 or 3 sf

(b)M1 Resolve vertically, *R* and *F* both resolved. Treat μmg as *F* for the first 5 marks.

A1 Correct equation. Treat μmg as *F*

M1 Resolve horizontally, *R* and *F* both resolved, acceleration in either form. Treat μmg as *F*

A1 For the lhs of the equation. Treat μmg as *F*

A1 For the rhs with acceleration as shown with 50 or *r*

ALT Parallel to the track: $F + mg\sin\theta = m\left(v^2/50\right)\cos\theta$ M1A1 50 or *r*

Perpendicular to the track: $R - mg\cos\theta = m\left(v^2/50\right)\sin\theta$ M1A1A1 50 or *r*

B1 $F = \dfrac{1}{4}R$ (μmg scores B0)

dM1 Eliminate *R* and *F* to obtain an equation for v^2. Depends on the two previous M marks, but if μmg used for *F* award M0 here.

A1 Correct value of *v*. Must be 2 or 3 sf.

Question Number	Scheme	Marks
7(a)	$\frac{1}{2}mv^2 - \frac{1}{2}m\times 10ag = mag$ $v^2 = 12ag$	M1A1
	After impact $V = e\sqrt{12ag}$	B1
	Energy to top: $\frac{1}{2}me^2(12ag) - \frac{1}{2}mW^2 = mag$	M1
	At top $T + mg = m\frac{W^2}{a}$	M1A1
	$T \geqslant 0 \quad W^2 \geqslant ag$	dM1
	$\frac{1}{2}me^2(12ag) - mag \left(= \frac{1}{2}mW^2\right) \geqslant \frac{1}{2}mag$	
	$\therefore e^2 \geqslant \frac{1}{4}, \quad \therefore e \geqslant \frac{1}{2}$ .... *	dM1,A1cso (9)
(b)	Energy to string breaking: $\frac{1}{2}m\frac{3}{4}(12ag) - \frac{1}{2}mX^2 = mag\cos 30°$	M1
	$X^2 = 9ag - 2ag\cos 30°$	A1
	Horiz speed $= X\cos 30° = \cos 30°\sqrt{ag(9 - 2\cos 30°)}$	M1
	By energy, speed at D = speed at B (after rebounding) $= \frac{\sqrt{3}}{2}\sqrt{12ag} \left(= 3\sqrt{ag}\right)$	M1
	OR use vert speed at C, $\left(X\sin 30° = \sin 30°\sqrt{ag(9 - 2\cos 30°)}\right)$, to find the vert speed at D by SUVAT $\cos\theta = \frac{\cos 30°\sqrt{ag(9 - 2\cos 30°)}}{3\sqrt{ag}}$ OR $\tan\theta = \frac{\text{vert speed}}{\text{horiz speed}}$	M1
	$\theta = \cos^{-1}\left(\frac{\cos 30°\sqrt{(9 - 2\cos 30°)}}{3}\right) = 38.89...°$	A1 (6)
	Accept 39º, 38.9º or better; or 0.68, 0.679, radians or better	[15]

Question Number	Scheme	Marks
(a)		
M1	Energy equation from *A* to *B*. Must have 2 KE terms and 1 PE term.	
A1	Fully correct equation	
B1	Correct speed immediately after impact.	
M1	Energy equation from *B* to *A*, with their speed after impact at *B*	
M1	Equation of motion along the radius at *A*	
A1	Correct equation at *A*	
dM1	Use tension at $A \geqslant 0$ to obtain a inequality for (speed at *A*)^2 Depends on the third M mark	
	ALT for the last 3 marks:	
	$m\frac{W^2}{a} - mg \geqslant 0$	
dM1	Use the energy equation to obtain $e^2 \geqslant ....$ Depends on second and third M marks	
A1cso	Reach the **given** answer with no errors seen	
(b)		
M1	Energy equation from *B* to *C* or from *A* to *C* $\left(\frac{3}{4}$ or e^2 used$\right)$	
A1	Correct expression for (speed at *C*)^2 Must have 3/4 now.	
M1	Attempt horizontal component of speed at *C* with their speed at *C*.	
M1	Obtain the speed at *D* or vertical speed at *D* by finding the vertical speed at *C* and using SUVAT . **NB: This is an A mark on e-PEN**	
M1	Use horizontal speed of *P* with (resultant) speed or vertical speed of *P* at *D* to form an expression for the cos or tan of the required angle.	
A1	Correct size of the angle in degrees or radians. 2 sf minimum (*g* cancels).	